Unpaired Majorana modes on dislocations in the Kitaev honeycomb model

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Outline

- Quantum computing
- Anyons
- Toric code
- Kitaev honeycomb model
- Dislocations in Kitaev honeycomb model

Quantum computing

data $\xrightarrow{\text{computation}}$ result

Use the principle of superposition:

$$\psi = \alpha|0\rangle + \beta|1\rangle$$

$$|\Psi_{initial}\rangle \longrightarrow |\Psi_{final}\rangle$$

Use the advantage of 2^n over n in going from n "bits" classical two level systems to n "qubits", the quantum two level system: Exponential Speed-up

Qubit: quantum two level system $\psi = |a||0\rangle + |b|e^{i\phi}|1\rangle$

Decoherence (the loss of information from a system into the environment) a central problem of quantum computation

Environment $\longrightarrow$ errors

TOPOLOGICAL Quantum computing

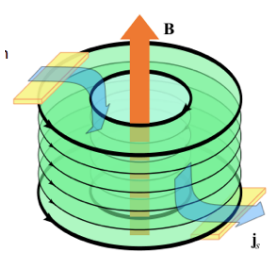
Idea due to Alexei Kitaev: If a physical system were to have quantum topological (necessarily nonlocal) degrees of freedom, which were insensitive to local probes, then information contained on them would be automatically protected against errors caused by local interactions with the environment.

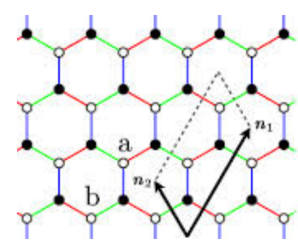
This would be fault tolerance guaranteed by physics at the hardware level, with no further need for quantum error correction, i.e. topological protection.

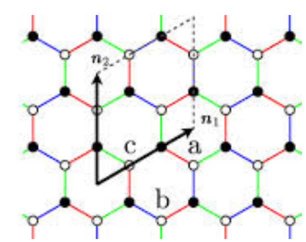
1. Topological quantum computation depends on the existence of non-Abelian topological phase.
2. The ability of manipulate quasiparticle excitations (anyons) in these phases

Topological QC: braiding of anyons = quantum gates

Topological superconductors, fractional quantum hall states, spin systems

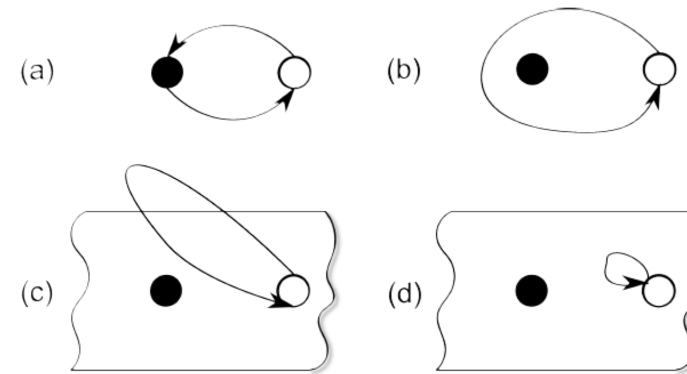






Anyons

Quantum statistics: behavior of under exchange of particles $\Psi(x_1, x_2, \ldots)$



In 3+1 dimensions, the only statistics are bosons and fermions. The exchange operation T can only be +/-1 since $T^2 = 1$. Therefore the system catch a phase such that

$$\Psi(x_1, x_2) \rightarrow T\Psi(x_1, x_2) = e^{i\phi}\Psi(x_1, x_2) = \pm\Psi(x_2, x_1)$$

In 2+1 dimensions $T^{-1} \neq T$, therefore the phase could be anything ➔ Anyons!

Abelian statistics $$\Psi_a \rightarrow e^{i\phi}\Psi_a$$

Non Abelian statistics $$\Psi_a \rightarrow T_{ab}\Psi_b$$

$$\Psi(x_1 \leftrightarrow x_3) = M \cdot \Psi(x_1, \ldots., x_n)$$

$$\Psi(x_2 \leftrightarrow x_3) = N \cdot \Psi(x_1, \ldots., x_n)$$

In general M and N do not commute

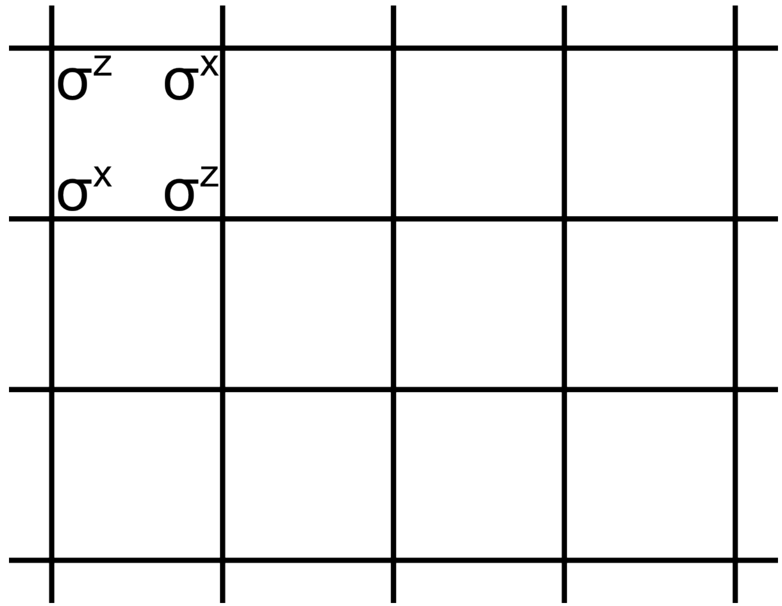
Majorana fermions

- Modification of the relativistic Dirac equation for particles with spin ½. Now the equation is real, it has real solution.
- therefore the particle will be its own antiparticle: $c_i = (c_i)^+$
- Electrically neutral.
- Half a fermion (real part of a Dirac (non-local) fermion)

$$c_i = \mathrm{Re}\,(\varepsilon) \qquad c_j = \mathrm{Im}\,(\varepsilon)$$

$$\varepsilon = c_i + i\, c_j \qquad \varepsilon^+ = c_i - i\, c_j$$

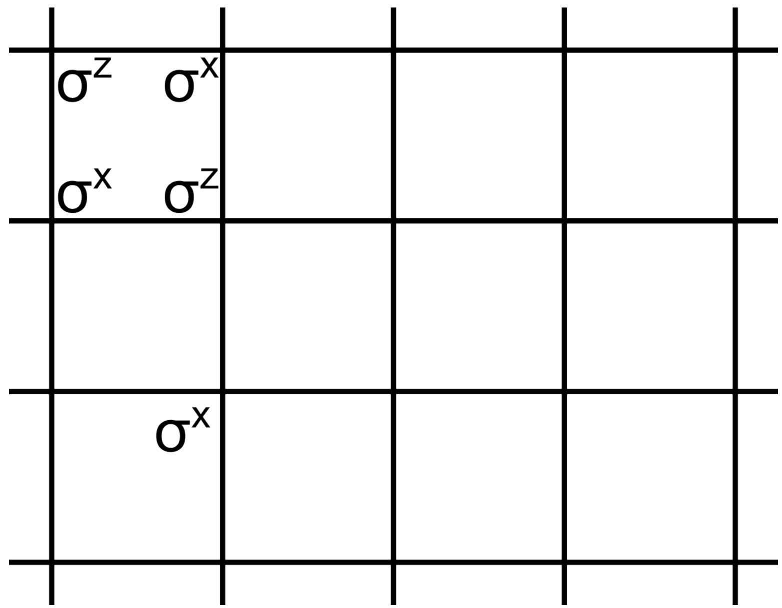
Toric Code



$$H = -\sum_p Q_p \qquad Q_p = \sigma_1^z \sigma_2^x \sigma_3^x \sigma_4^z$$

X-G. Wen, Phys. Rev. Lett. 90, 016803 (2003)

Toric Code, creating excitations



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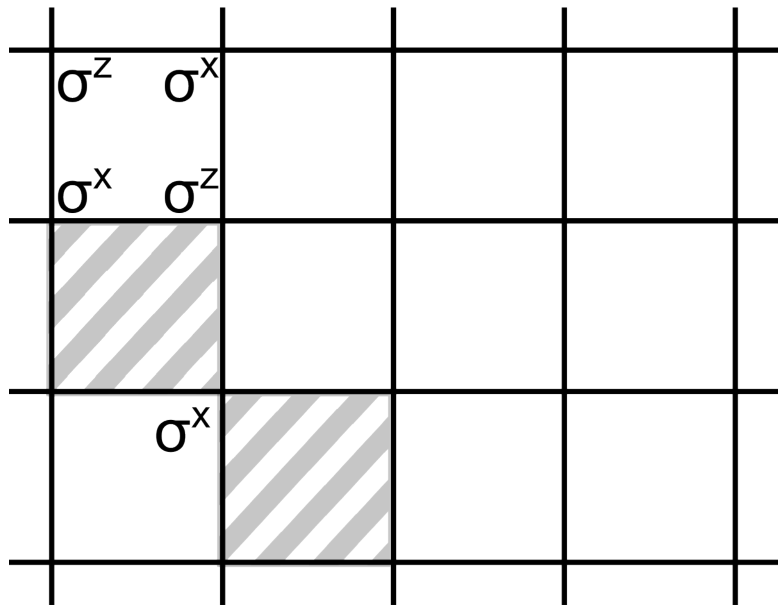

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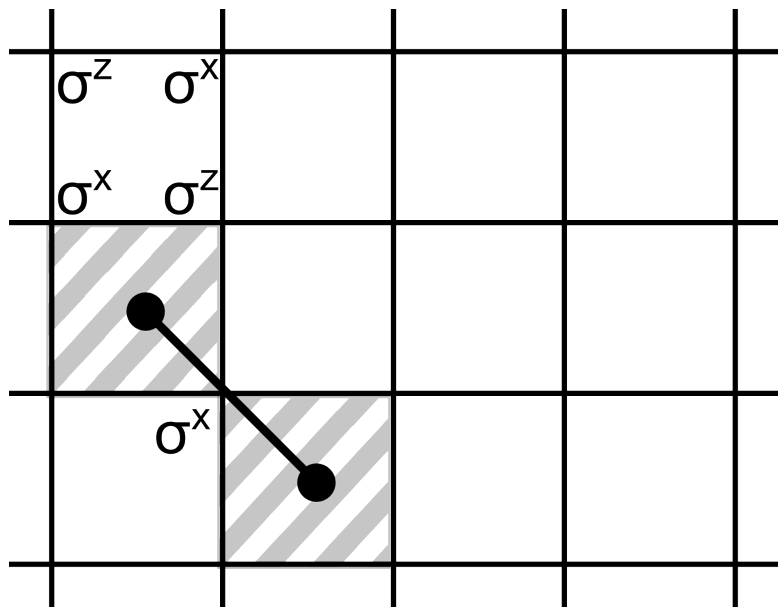

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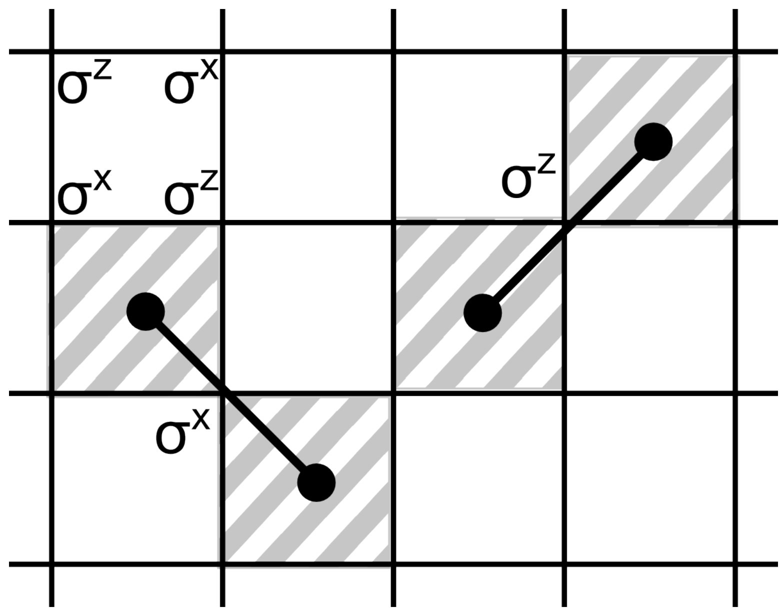

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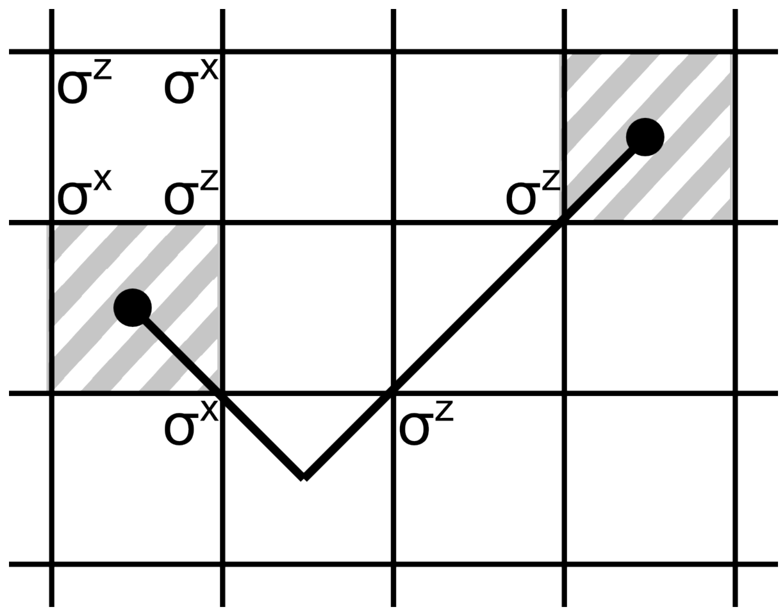

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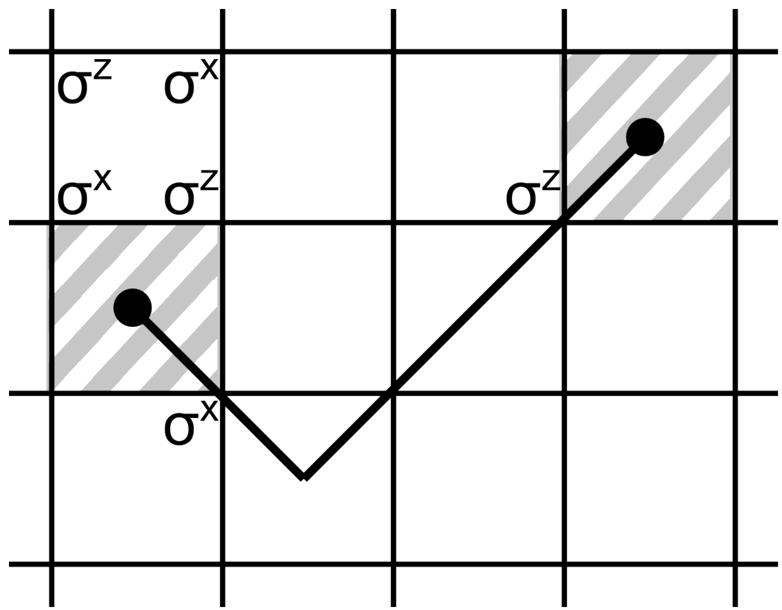
Toric Code

strings are invisible, only the ends are fixed



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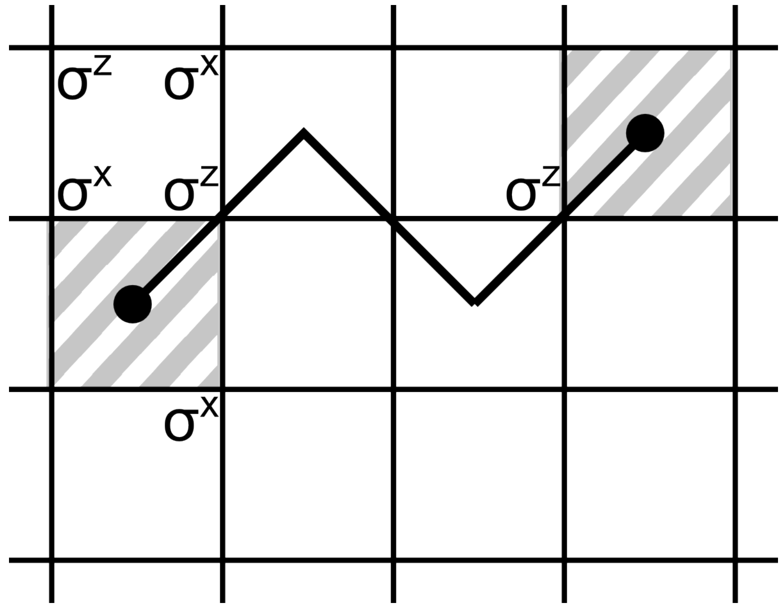

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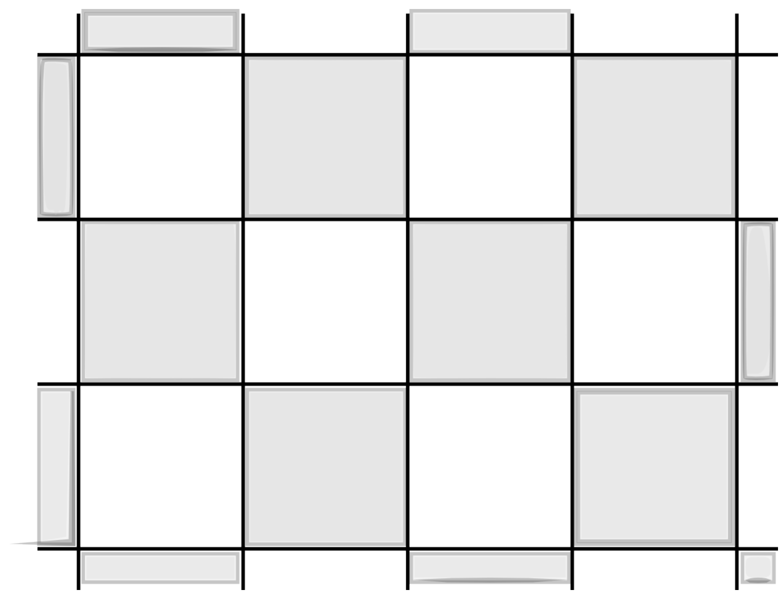

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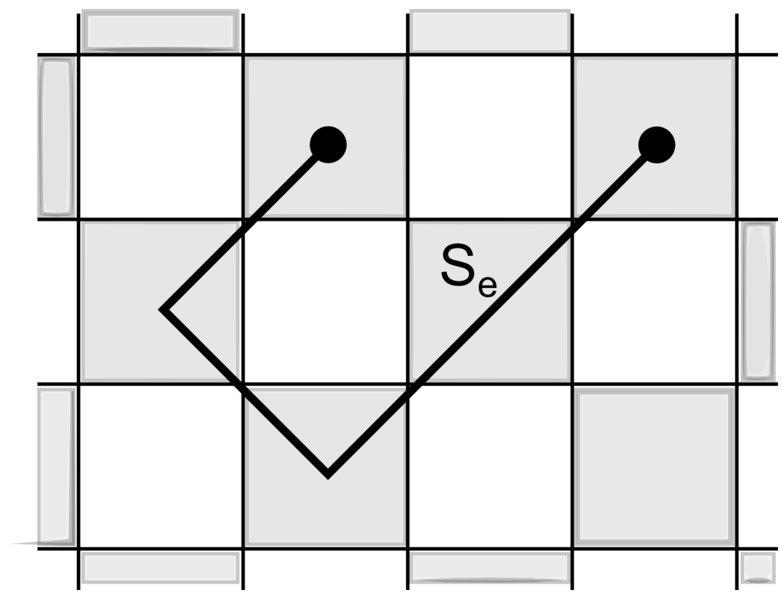

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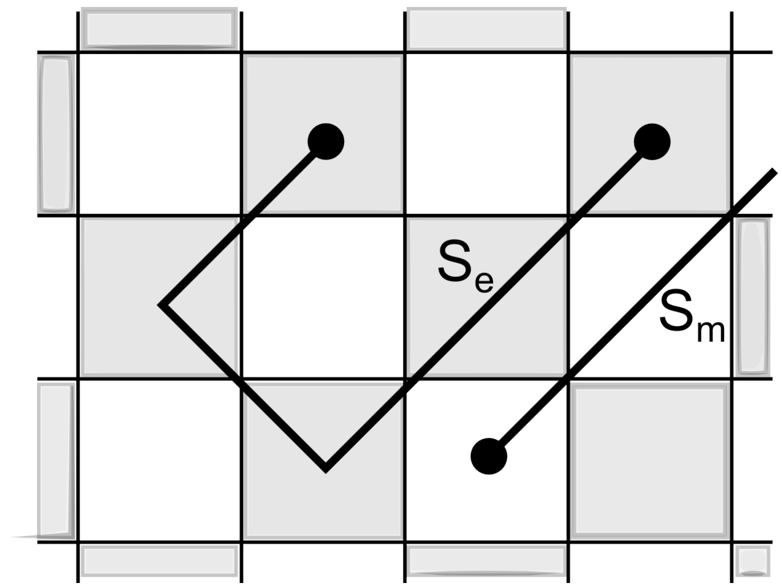

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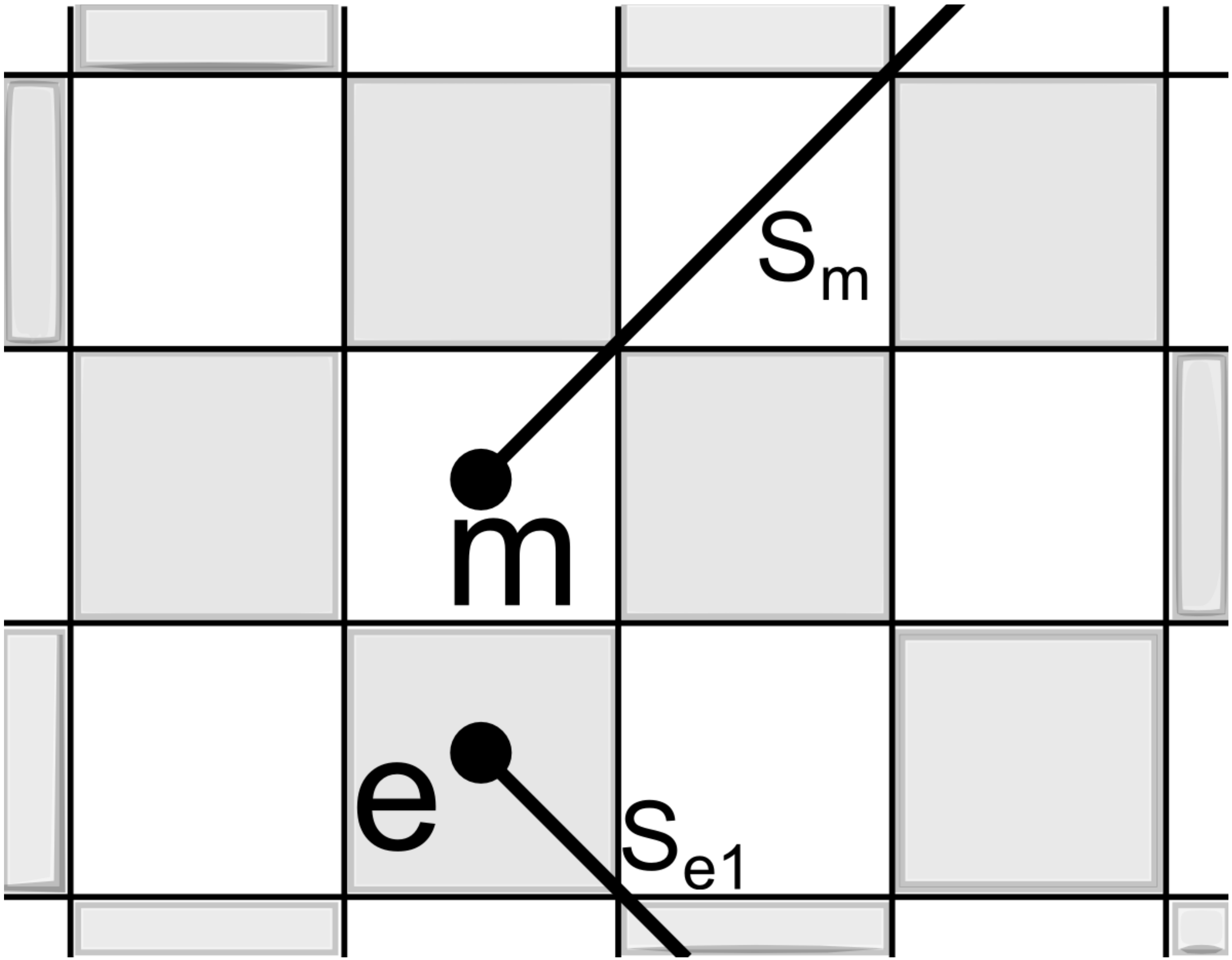

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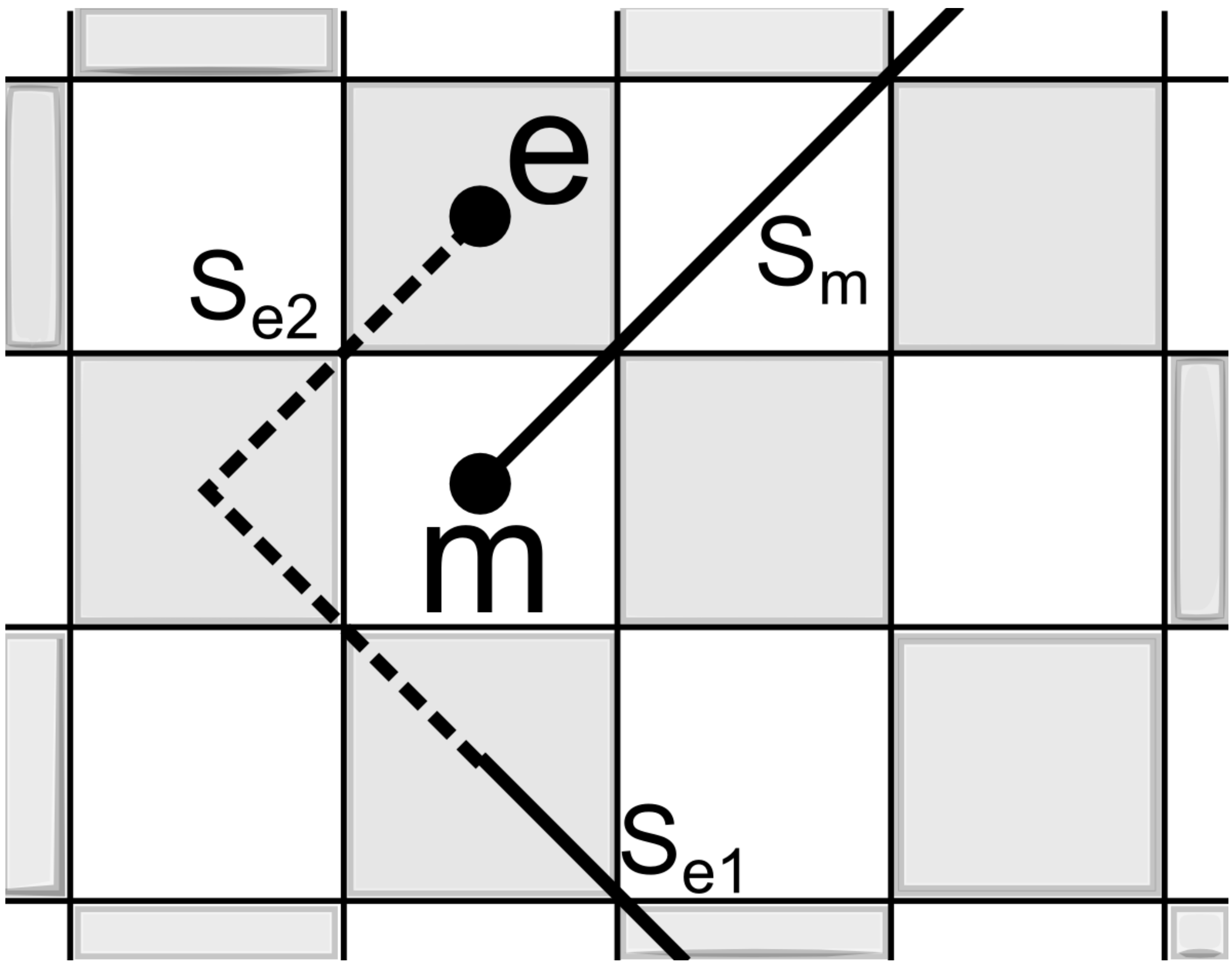
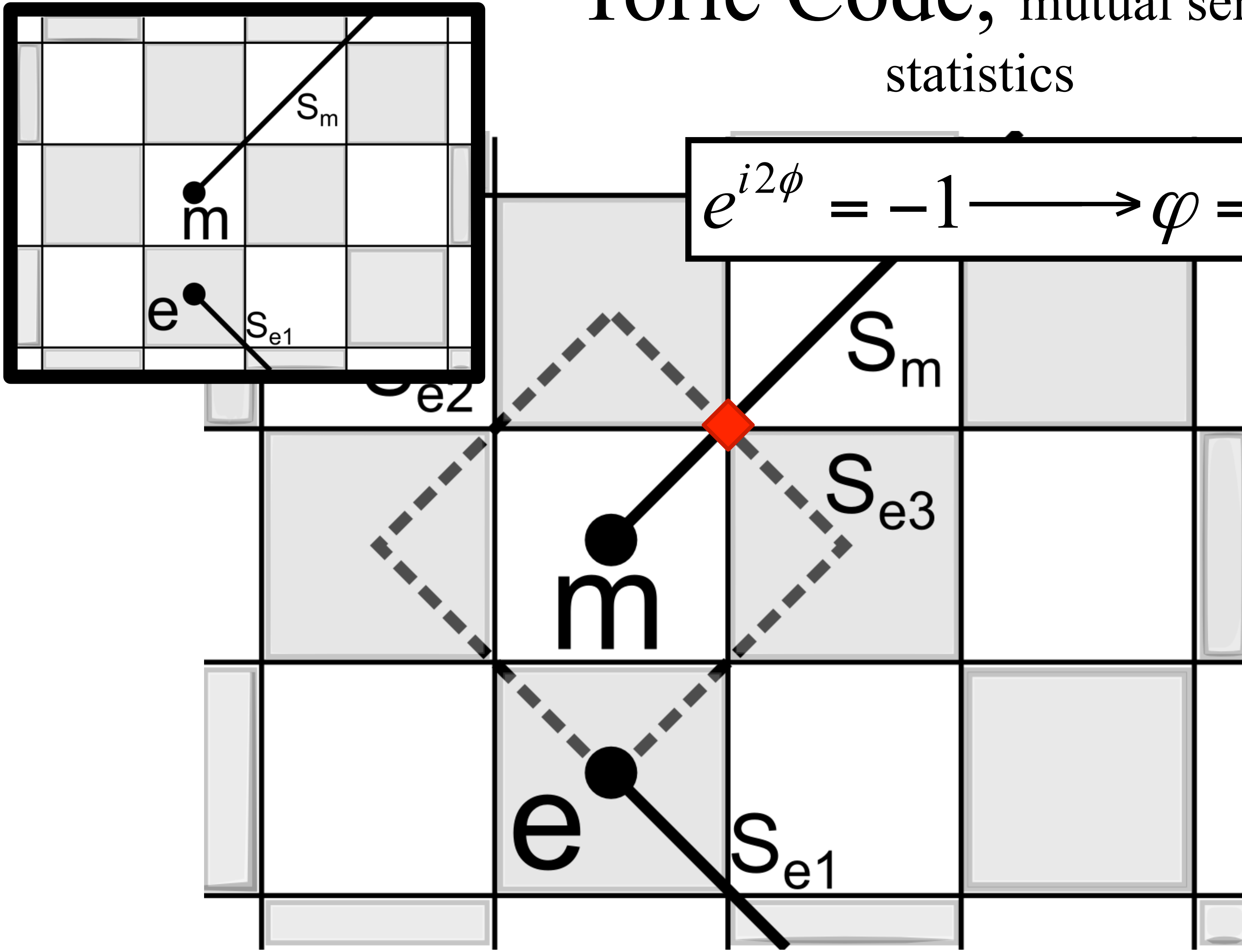


Toric Code, mutual semionic statistics

$$e^{i2\phi} = -1 \longrightarrow \varphi = \pi/2$$



Toric Code

- e: boson
- m: boson
- e and m: mutual semions ($\varphi = \pi/2$)
- Composite e x m = ε is a fermion

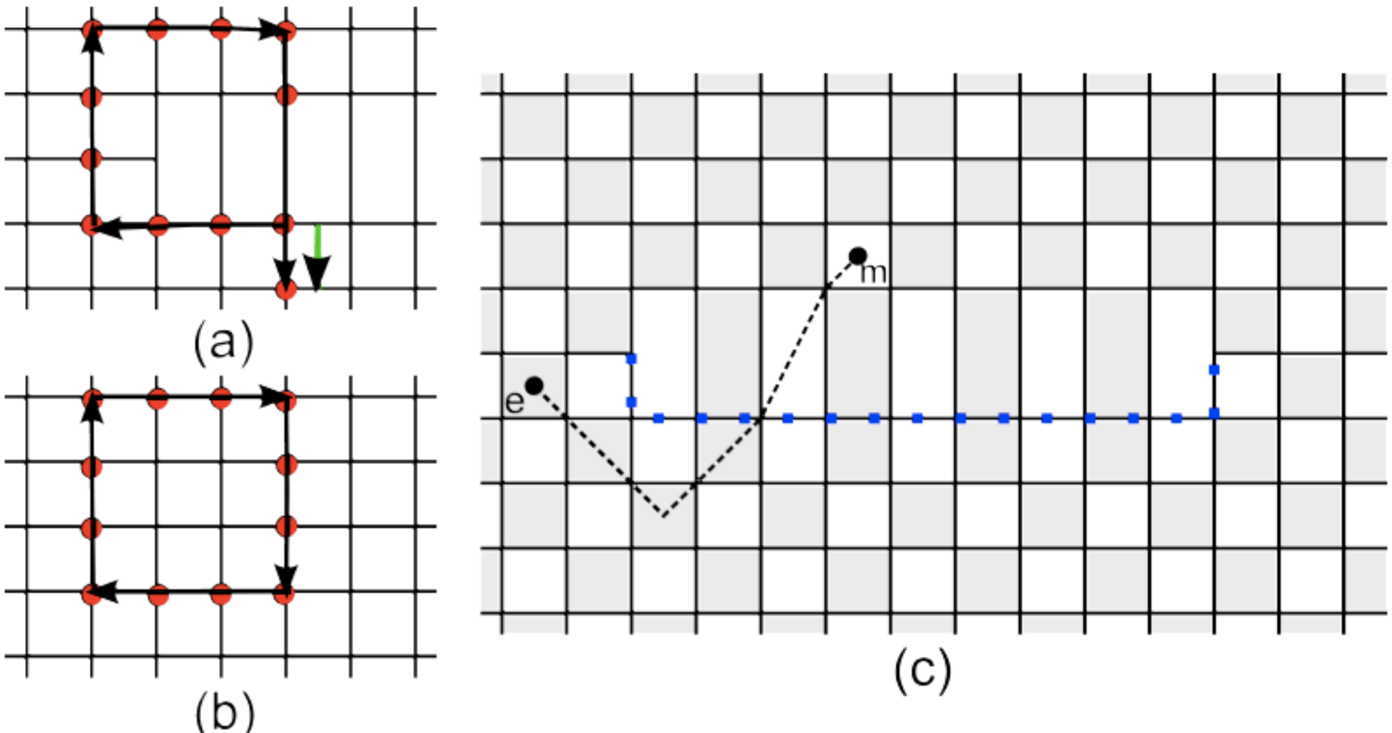
Since one can only create pairs of e and pairs of m charges in the toric code, we cannot create an odd number of fermions out of vacuum. Parity is conserved

The fusion rules, which describe the outcome of joining two particles, are:

$$e \times e = m \times m = 1, e \times m = \epsilon$$

Twist defects in the toric code

The toric code has three topological kinds of particles (in addition to the vacuum): electric charge e, magnetic flux m, and a composite fermion .

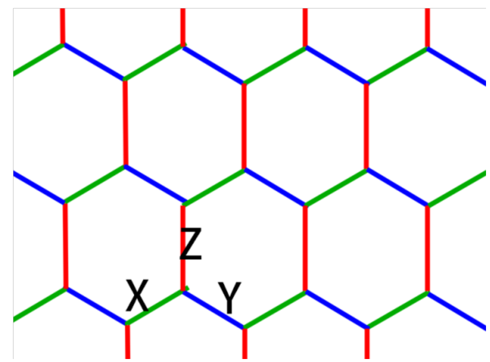


$$\sigma \times \sigma = 1 + \epsilon, \sigma \times \epsilon = \sigma, \epsilon \times \epsilon = 1$$

One can show, that each dislocation has the braiding and fusion rules of an Ising anyon

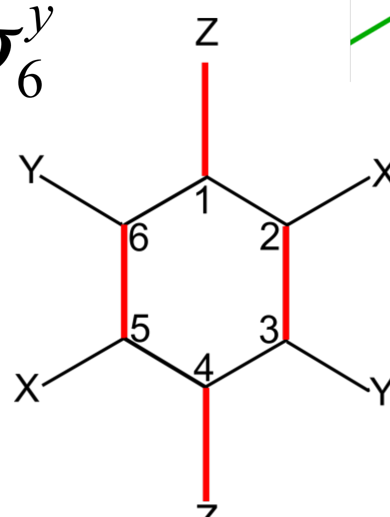
Kitaev Honeycomb Model

$$H = -J_x \sum_x \sigma^x_m \sigma^x_n - J_y \sum_y \sigma^y_m \sigma^y_n - J_z \sum_z \sigma^z_m \sigma^z_n$$



Conserved quantities, plaquette flux:

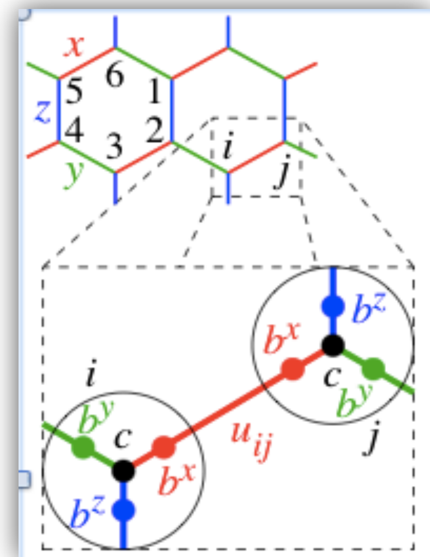
$$W_p = \sigma_1^z \sigma_2^x \sigma_3^y \sigma_4^z \sigma_5^x \sigma_6^y$$



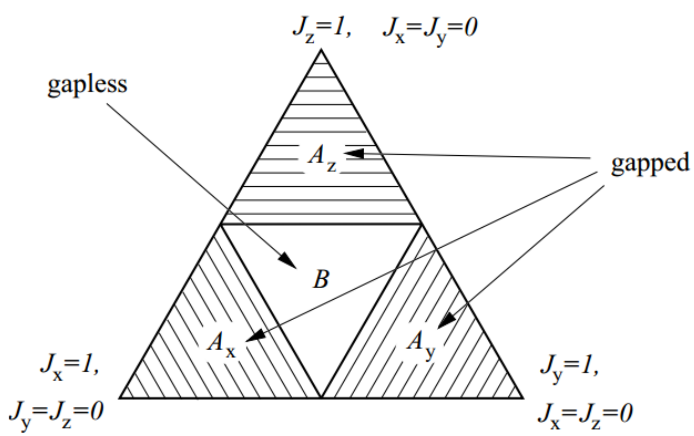
A. Kitaev, Annals of Physics 321, 2 (2006)

From spins to Majoranas

$$\sigma^{\alpha} = i b^{\alpha} c \qquad \alpha = x, y, z$$



$$H = i \Sigma_{\langle m,n \rangle} J_{\alpha_{mn}} u_{mn} c_m c_n$$



A. Kitaev, Annals of Physics 321, 2 (2006)

$$u_{mn} = i b_m^{\alpha} b_n^{\alpha}$$

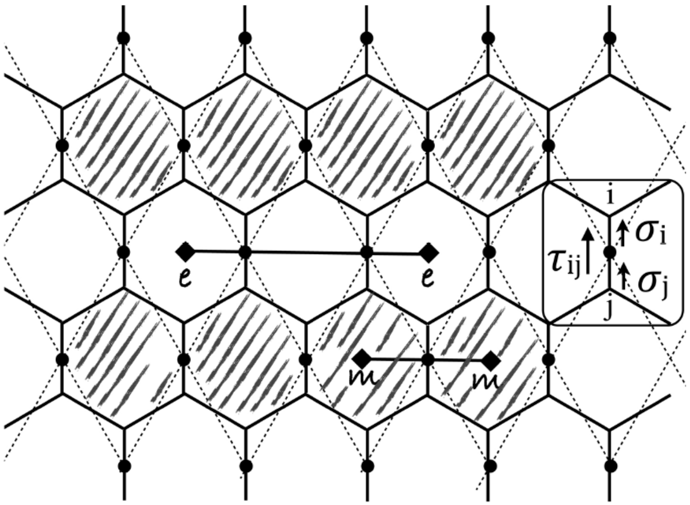
$$W = (-u_{12}) u_{23} (-u_{34}) u_{45} (-u_{56}) u_{61}$$

is a flux of the Z_2 gauge field. Factor -1 for each link pointing from the odd to the even sublattice.

$$H = \sum_m \frac{\epsilon_m}{2} \left(\psi_m^{\dagger} \psi_m - \psi_m \psi_m^{\dagger} \right)$$

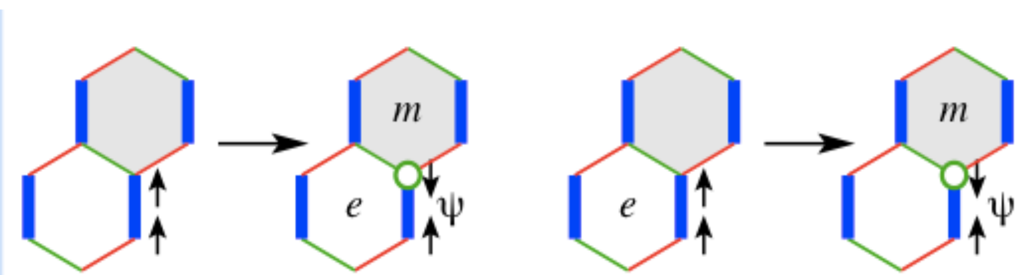
in its diagonal form. In the **GS** sector $W = 1$

From spins to Majoranas

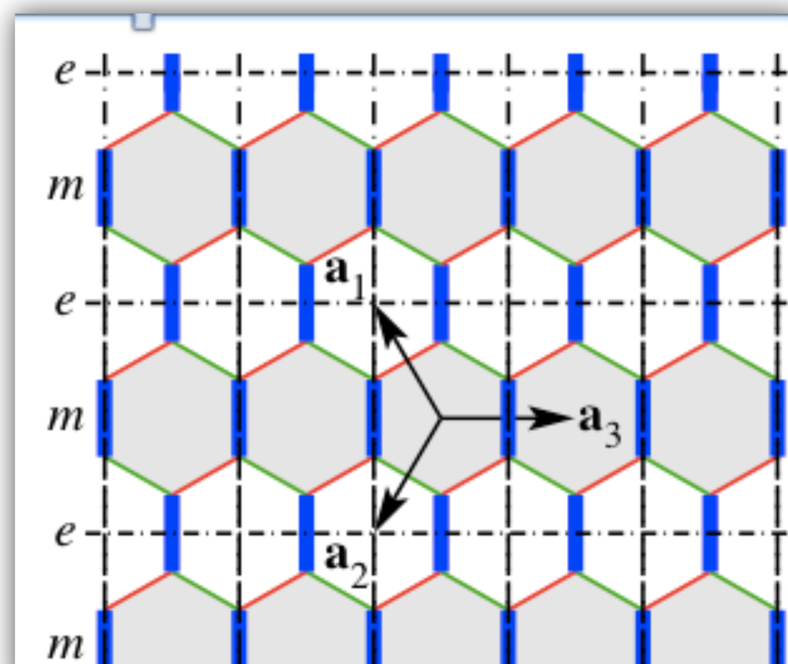


Low-energy excitations are vortices, $W = -1$, with energy $\dfrac{J_x^2 J_y^2}{8 J_z^3}$

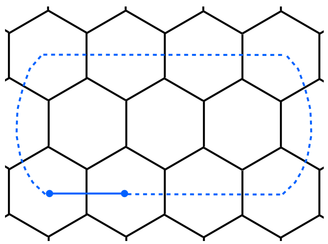
$$H = -\sum_{x\text{-links}} J_x \sigma_i^x \sigma_j^x - \sum_{y\text{-links}} J_y \sigma_i^y \sigma_j^y - \sum_{z\text{-links}} J_z \sigma_i^z \sigma_j^z$$



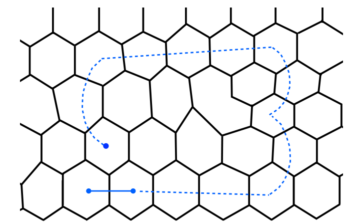
In the gapped phase $J_z \gg J_x, J_y$. High-energy excitations cost $2J_z$. They are fermion excitations, associated with breaking a strong bond.



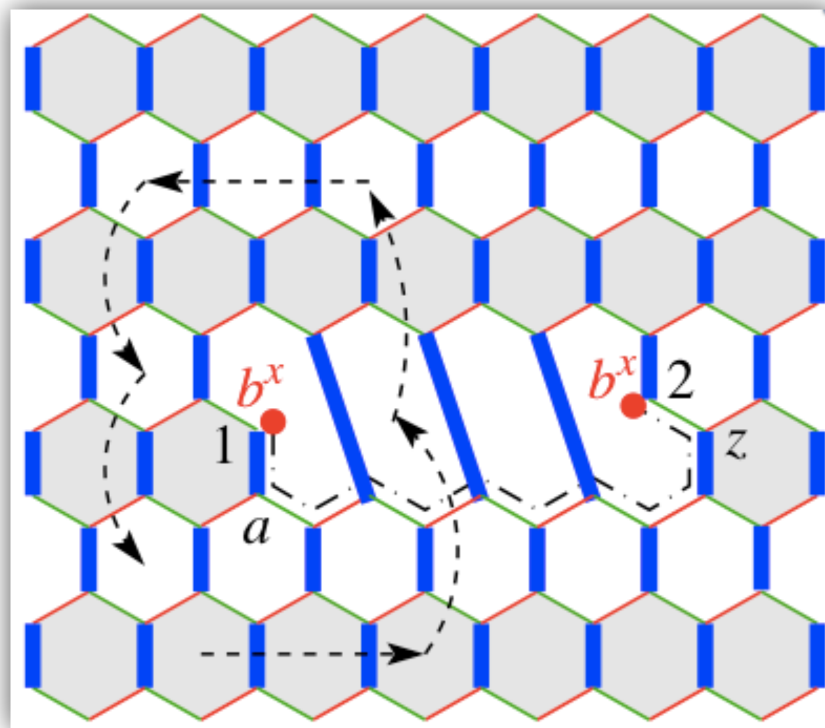
Dislocations in the honeycomb lattice



A dislocation in Kitaev honeycomb model is made up of an octagon and a site whose coordination number is 2 instead of 3,



Presence of a dislocation in a lattice can be identified by a non-zero Burgers vector.



local fermion, whose parity is given by

$$\pi_d = i \prod_{(ij)\in C} u_{ij} b_1^x b_2^x$$

Summary

Each dislocation can be associated with an unpaired Majorana mode, which for a dislocation pair can be combined into a complex fermion.

Non-Abelian statistics may arise from an Abelian phase of the Kitaev honeycomb model in the presence of dislocations.